2020

MATHEMATICS (General)

Paper : MTMG - DC-1/GE-1

[CBCS]

Full Marks : 32 Time : Two Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

Group - A

1. Answer any **four** questions. $1 \times 4 = 4$

   (a) For what values of λ, the equation $x^2 + \lambda xy - 2y^2 + 3y - 1 = 0$ represents a pair of straight lines?

   (b) Show that if $a \mid b$ and $a \mid c$, then $a \mid (b + c)$.

   (c) Find the complex conjugate of $\frac{2+3i}{5+7i}$.

   (d) Find an equation of degree 3 with real coefficient having roots 2 and $2 + 3i$.

   (e) Show that the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ is orthogonal.

   (f) Find the pole of the line $x+2y+3 = 0$ with respect to the circle $x^2+y^2-2x+5 = 0$.

   (g) Find the equation of tangent(s) at the origin to the curve $2x^2+5xy+3y^2+4x-10y = 0$.

Group - B

Answer any *two* questions. 5×2=10

2. If $u + iv = \operatorname{cosec}(x + iy)$, then show that

$$(u^2 + v^2)^2 = \frac{u^2}{\sin^2 x} - \frac{v^2}{\cos^2 x} = \frac{u^2}{\cosh^2 y} + \frac{v^2}{\sinh^2 y}. \quad [5]$$

3. If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, then find the value of $(\alpha + \beta)(\beta + \gamma)(\gamma + \alpha)$. [5]

4. Prove that the feet of the normals from the point (α, β) to the parabola $y^2 = 4ax$ lie on the curve $xy - (\alpha - 2a)\, y = 2a\beta$. [5]

5. Find the general equation of a conic in polar form (r, θ), with focus as a pole. [5]

Group - C

Answer any *two* questions. 9×2=18

6. (a) If one of the straight lines given by the equation $ax^2 + 2hxy + by^2 = 0$ coincides with one of the straight lines given by $a'x^2 + 2h'xy + b'y^2 = 0$ and the remaining two straight lines are at right angles, then show that $h\left(\frac{1}{b} - \frac{1}{a}\right) = h'\left(\frac{1}{b'} - \frac{1}{a'}\right)$. [5]

   (b) Reduce the equation $x^2 + 4y^2 - 20x + 40y + 196 = 0$ to its canonical form. [4]

7. (a) Find the equations of the planes parallel to the plane $16x + 12y - 15z + 75 = 0$ and at a distance 4 from it. [5]

   (b) Find the image of the point $(1, -2, 3)$ in the plane $2x - 3y + 2z + 3 = 0$. [4]

8. (a) Reduce the following matrix to row reduced echelon matrix and also find its rank

$$\begin{pmatrix} 1 & 3 & 4 & 5 \\ 1 & 2 & 5 & 7 \\ 1 & 5 & 0 & 1 \end{pmatrix}. \quad [5]$$

   (b) Show that

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3. \quad [4]$$