UG/6th Sem (G)/22 (CBCS)

U.G. 6th Semester Examinations 2022

MATHEMATICS (General)

Paper Code : DSE - 2-A & B

[CBCS]

Full Marks : 32 | Time : Two Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Paper Code : DSE - 2-A

[REAL & COMPLEX ANALYSIS]

Group - A

(4 Marks)

1. Answer any ***four*** questions : 1×4=4

(a) Solve the equation : $|z| - z = 2 + i$

(b) Define analytic function.

(c) Find the radius of convergence of the power series $1 + 3x + \frac{3^2}{\lfloor 2}x^2 + \frac{3^3}{\lfloor 3}x^3 + ...$

(d) State convolution theorem for Laplaces transformation.

(e) Find $\mathscr{L}^{-1}\left(\frac{1}{S^2+7}\right); S > 0$.

(f) Define uniform convergence of a sequence of functions $\{f_n\}$ on an interval I.

(g) Write down the statement of Dirichlet's conditions of convergence.

Group - B

(10 Marks)

Answer any ***two*** questions : 5×2=10

2. For the function $f(z)$, defined by

$$f(z)=\begin{cases}\dfrac{(\overline{z})^2}{z} & , z \neq 0 \\ 0 & , z=0\end{cases}$$

Show that the *C–R* equations are satisfied at (0, 0) but the function is not differentiable at 0 + 0*i*. 5

3. Find the Fourier series consisting of sine terms only, which represents the periodic function $f(x)=x$ in $0 \leq x \leq \pi$. 5

4. For each natural number *n,* let $f_n(x)=\dfrac{x}{1+nx^2}$, $x \in [0,1]$, show that the sequence of function $\{t_n\}_n$ converge uniformly on [0, 1].

5. Use Laplace transform to solve the initial value problem.

$y''+3y'+2y=e^{-t}$, $y(0)=y'(0)=0$ 5

Group - C

(18 Marks)

Answer any ***two*** questions : 9×2=18

6. (a) Show that the sequence $\{f_n\}$, where $f_n(x)=\tan^{-1} nx$, $x \geq 0$ is uniformly convergent in any interval [*a, b*], *a* > 0, but is only point-wise convergent in [0, b]. 7

   (b) Show that the function $u(x,y)=2x-x^3+3xy^2$ is a harmonic function. 2

7. (a) Prove that the function $f(z)=|z|$ is no where differentiable in *C* but continuous everywhere. 7

   (b) Find the set of points for which the function $f(z)=\dfrac{z-1}{z^2+1}$ is not analytic. 2

8. (a) Obtain the Fourier series in $[-\pi, \pi]$ for the function $f(x) = x \sin x.\ -\pi \le x \le \pi$. 7

(b) Find the Laplace transform, if it exists for the function $f(t) = e^{at}$. 2

Paper Code : DSE - 2 (2)

[LINEAR PROGRAMMING PROBLEM & GAME THEORY]

Group - A

(4 Marks)

1. Answer any ***four*** questions : 1×4=4

(a) Show that the set $X = \{x : |x| \le 2\}$ is a Convex set.

(b) A hyperplane is given by $x_1 + 2x_2 + 5x_3 + 2x_4 = 2$. Find in which half spaces the point (1, 2, –3, 1) lies.

(c) Write down the dual of the following Problems :

Maximize $Z = 4x_1 + 2x_2$

Subject to $3x_1 + 4x_2 \le 7$

$7x_1 - 2x_2 \le 13,\ x_1 \ge 0,\ x_2 \ge 0$

(d) What is the convex hull of the set $S = \{(x, y) : x^2 + y^2 = 4\}$.

(e) Find out the extreme points of the following convex set :

$S = \{(x, y) \mid x^2 + y^2 \le 25\}$

(f) Define basic feasible solution.

(g) Solve the game Problem and determine the value of the game

Player B

Player A	2	5
	7	3

Group - B

(10 Marks)

Answer any ***two*** questions : 5×2=10

2. Solve graphically the L.P.P.

Minimize $Z = 2x_1 + 3x_2$

Subject to $2x_1 + 7x_2 \geq 22$

$x_1 + x_2 \geq 6$

$5x_1 + x_2 \geq 10, \quad x_1, x_2 \geq 0$

3. Find out an initial B.F.S. of the following balanced T.P using row minima method

	D_1	D_2	D_3	D_4	a_i	
O_1	4	2	5	3	6	
O_2	5	4	3	2	13	Capasity
O_3	1	4	6	5	9	
b_j	7	8	5	8		

Demand 5

4. Prove that $x_1 = 2$, $x_2 = 1$ and $x_3 = 3$ is a feasible solution of the set of equations

$$4x_1 + 2x_2 - 3x_3 = 1$$

$$-6x_1 - 4x_2 + 5x_3 = -1$$

Reduce the feasible solution to a basic feasible solution by reduction theory. 5

5. Solve the following game problem graphically : 5

$$\begin{bmatrix} 1 & 2 & -3 & 7 \\ 2 & 5 & 4 & -6 \end{bmatrix}$$

Group - C

(18 Marks)

Answer any ***two*** questions : 9×2=18

6. (a) Find the optimal assignments to find the minimum cost for the assignment problem with the cost matrices :

	I	II	III	IV	V
A	6	5	8	11	16
B	1	13	16	1	10
C	16	11	8	8	8
D	9	14	12	10	16
E	10	13	11	8	16

6

(b) How many basic solutions are there in the following set of equations. Find all the basic solutions of the system of equations :

$$x_1 + x_2 + 2x_3 = 9$$
$$3x_1 + 2x_2 + 5x_3 = 22$$ 3

7. (a) Rewrite the L.P.P. in standard maximization form by supplying slack and surplus variables

Minimize $Z = 3x_1 - 2x_2 + 4x_3$

Subject to $x_1 - x_2 + 3x_3 \geq 1$

$2x_1 + 3x_2 - 5x_3 \geq -3$

$4x_1 + 2x_2 \geq 2,\ x_1, x_2, x_3 \geq 0$

State which are the slack and surplus variable. 3

(b) Use duality to solve the L.P.P.

Minimize $Z = 3x_1 + x_2$

Subject $2x_1 + 3x_2 \geq 2$

$x_1 + x_2 \geq 1,\quad x_1, x_2 \geq 0$ 6

8. (a) Solve the following game stating the optimal strategies and the saddle points :

$$\begin{bmatrix} 2 & 3 & 2 & 4 & 6 \\ 0 & -2 & 1 & 2 & 1 \\ -1 & 3 & 0 & -1 & 3 \\ 4 & 5 & -1 & 2 & 1 \\ 3 & 2 & -2 & 1 & -2 \end{bmatrix}$$ 5

(b) Solve the following L.P.P. by Big M-method :

Minimize $Z = 2x_1 + 9x_2 + x_3$

Subject to $x_1 + 4x_2 + 2x_3 \geq 5$

$3x_1 + x_2 + 2x_3 \geq 4$

and $x_1, x_2, x_3 \geq 0$ 4
